Data structures and libraries

Computer Science Enrichment Club - Algorithms Division

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Today we're going to cover

- Basic data types
- Big integers
- Why we need data structures
- Data structures you already know
- Sorting and searching
- Using bitmasks to represent sets
- Common applications of the data structures
- Augmenting binary search trees
- Representing graphs

Basic data types

- You should all be familiar with the basic data types:
  - `bool`: a boolean (`true`/`false`)
  - `char`: an 8-bit signed integer (often used to represent characters with ASCII)
  - `short`: a 16-bit signed integer
  - `int`: a 32-bit signed integer
  - `long long`: a 64-bit signed integer
  - `float`: a 32-bit floating-point number
  - `double`: a 64-bit floating-point number
  - `long double`: a 128-bit floating-point number
  - `string`: a string of characters

Basic data types

Type	Bytes	Min value	Max value
bool	1		
char	1	-128	127
short	2	-32768	32767
int	4	-2148364748	2147483647
long long	8	-9223372036854775808	9223372036854775807
	n	$-2^{\mathbf{8}n-\mathbf{1}}$	$2^{\mathbf{8}n-\mathbf{1}}-1$

Type	Bytes	Min value	Max value
unsigned char	1	0	255
unsigned short	2	0	65535
unsigned int	4	0	4294967295
unsigned long long	8	0	18446744073709551615
	n	0	$2^{\mathbf{8}n}-1$

Type	Bytes	Min value	Max value	Precision
float	4	$\approx -3.4 \times 10^{\mathbf{38}}$	$\approx 3.4 \times 10^{\mathbf{38}}$	$\approx$ 7 digits
double	8	$\approx -1.7 \times 10^{\mathbf{308}}$	$\approx 1.7 \times 10^{\mathbf{308}}$	$\approx$ 14 digits
long double	16	$\approx -1.1 \times \times 10^{\mathbf{4932}}$	$\approx 1.1 \times 10^{\mathbf{4932}}$	$\approx$ 18 digits

Big integers

- What if we need to represent and do computations with very large integers, i.e. something that doesn't fit in a `long long`
- Simple idea: Store the integer as a string
- But how do we perform arithmetic on a pair of strings?
- We can use the same algorithms as we learned in elementary school
  - Addition: Add digit-by-digit, and maintain the carry
  - Subtraction: Similar to addition
  - Multiplication: Long multiplication
  - Division: Long division
  - Modulo: Long division

Example problem: Simple Addition

- https://open.kattis.com/problems/simpleaddition

Why do we need data structures?

- Sometimes our data needs to be organized in a way that allows one or more of
  - Efficient querying
  - Efficient inserting
  - Efficient deleting
  - Efficient updating
- Sometimes we need a better way to represent our data
  - How do we represent large integers?
  - How do we represent graphs?
- Data structures help us achieve those things

Data structures you've seen before

- Static arrays
- Dynamic arrays
- Linked lists
- Stacks
- Queues
- Priority queues
- Sets
- Maps

Data structures you've seen before

- Static arrays - `int arr[10]`
- Dynamic arrays - `vector<int>`
- Linked lists - `list<int>`
- Stacks - `stack<int>`
- Queues - `queue<int>`
- Priority queues - `priority_queue<int>`
- Sets - `set<int>`
- Maps - `map<int, int>`

Data structures you've seen before

- Static arrays - int arr[10]
- Dynamic arrays - vector<int>
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- Queues - queue<int>
- Priority queues - priority_queue<int>
- Sets - set<int>
- Maps - map<int, int>

- Usually it's best to use the standard library implementations
  - Almost surely bug-free and fast
  - We don't need to write any code
- Sometimes we need our own implementation
  - When we want more flexibility
  - When we want to customize the data structure

Sorting and searching

- Very common operations:
  - Sorting an array
  - Searching an unsorted array
  - Searching a sorted array
- Again, usually in the standard library
- We'll need different versions of binary search later which need custom code, but lower_bound is enough for now

Sorting and searching

- Very common operations:
  - Sorting an array - sort(arr.begin(), arr.end())
  - Searching an unsorted array - find(arr.begin(), arr.end(), x)
  - Searching a sorted array - lower_bound(arr.begin(), arr.end(), x)
- Again, usually in the standard library
- We'll need different versions of binary search later which need custom code, but lower_bound is enough for now

Representing sets

- We have a small ($n \leq 30$) number of items
- We label them with integers in the range $0, 1, \ldots, n-1$
- We can represent sets of these items as a 32-bit integer
- The ith item is in the set represented by the integer x if the ith bit in x is 1
- Example:
  - We have the set $\{0, 3, 4\}$
  - `int x = (1<<0) | (1<<3) | (1<<4);`

Representing sets

- Empty set:

  `0`

- Single element set:

  `1<<i`

- The universe set (i.e. all elements):

  `(1<<n)-1`

- Union of sets:

  `x|y`

- Intersection of sets:

  `x&y`

- Complement of a set:

  `~x & ((1<<n)-1)`

Representing sets

- Check if an element is in the set:

```
if (x & (1<<i)) {
    // yes
} else {
    // no
}
```

Representing sets

- Why do this instead of using set<int>?
- Very lightweight representation
- All subsets of the n elements can be represented by integers in the range $0 \ldots 2^n - 1$
- Allows for easily iterating through all subsets (we'll see this later)
- Allows for easily using a set as an index of an array (we'll see this later)

Applications of representing sets

- Most problems require storing data, usually in an array
- Makes a some problems simpler.
- http://wcipeg.com/problem/ccc11s5 - solution
- http://wcipeg.com/problem/dwitesep09p4 - solution

Applications of Arrays and Linked Lists

- Too many to list
- Most problems require storing data, usually in an array

Applications of Stacks

- Processing events in a last-in first-out order
- Simulating recursion
- Depth-first search in a graph
- Reverse a sequence
- Matching brackets
- And a lot more

Example problem: Backspace

- https://open.kattis.com/problems/backspace

Applications of Queues

- Processing events in a first-in first-out order
- Breadth-first search in a graph
- And a lot more

Applications of Priority Queues

- Processing events in order of priority
- Finding a shortest path in a graph
- Some greedy algorithms
- And a lot more

Applications of Sets

- Keep track of distinct items
- Have we seen an item before?
- If implemented as a binary search tree:
    - Find the successor of an element (the smallest element that is greater than the given element)
    - Count how many elements are less than a given element
    - Count how many elements are between two given elements
    - Find the kth largest element
- And a lot more

Applications of Maps

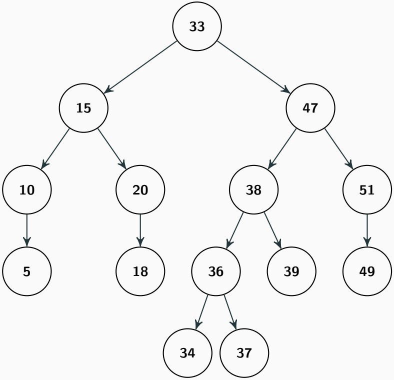
- Associating a value with a key
- As a frequency table
- As a memory when we're doing Dynamic Programming (later)
- And a lot more

Augmenting Data Structures

- Sometimes we can store extra information in our data structures to gain more functionality
- Usually we can't do this to data structures in the standard library
- Need our own implementation that we can customize
- Example: Augmenting binary search trees

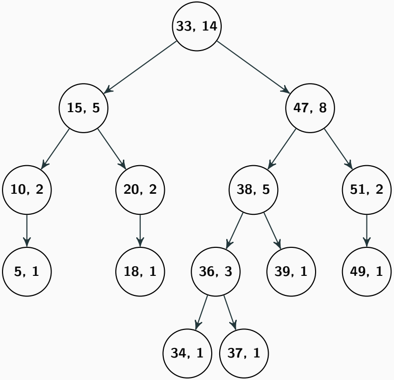
Augmenting Binary Search Trees

- We have a binary search tree and want to efficiently:
    - Count number of elements $< x$
    - Find the kth smallest element
- Naive method is to go through all vertices, but that is slow: $O(n)$



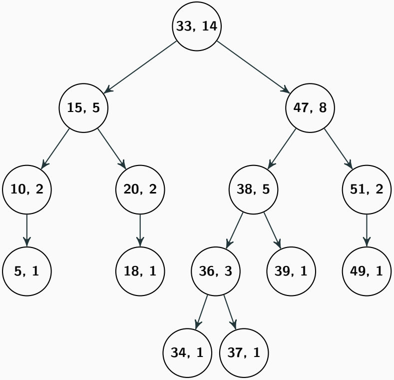
Augmenting Binary Search Trees

- Idea: In each vertex store the size of the subtree
- This information can be maintained when we insert/delete elements without increasing time complexity



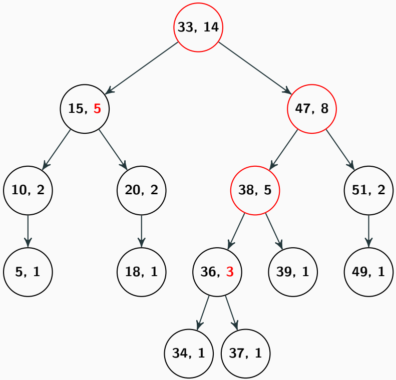
Augmenting Binary Search Trees

- Count number of elements < 38
  - Search for 38 in the tree
  - Count the vertices that we pass by that are less than x
  - When we are at a vertex where we should go right, get the size of the left subtree and add it to our count



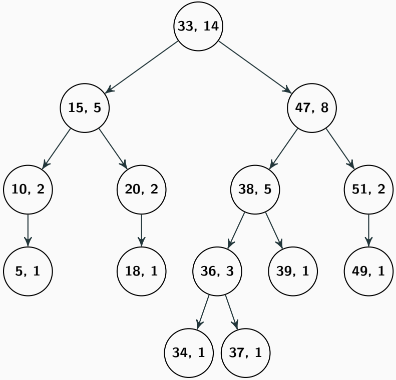
Augmenting Binary Search Trees

- Count number of elements < 38
  - Search for 38 in the tree
  - Count the vertices that we pass by that are less than x
  - When we are at a vertex where we should go right, get the size of the left subtree and add it to our count
- Time complexity $O(\log n)$



Augmenting Binary Search Trees

- Find kth smallest element
  - We're on a vertex whose left subtree is of size m
  - If $k = m + 1$, we found it
  - If $k \leq m$, look for the kth smallest element in the left subtree
  - If $k > m + 1$, look for the $k - m - 1$st smallest element in the right subtree



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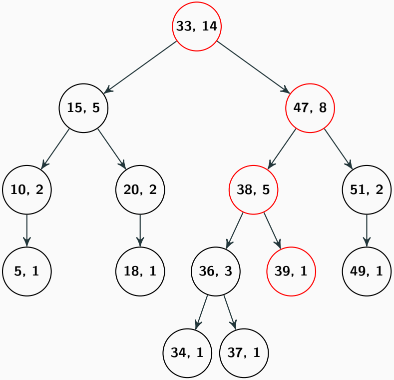
- Example: $k = 11$



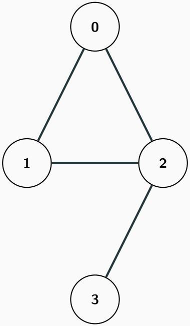
Representing graphs

- There are many types of graphs:
    - Directed vs. undirected
    - Weighted vs. unweighted
    - Simple vs. non-simple
- Many ways to represent graphs
- Some special graphs (like trees) have special representations
- Most commonly used (general) representations:
    1. Adjacency list
    2. Adjacency matrix
    3. Edge list

Adjacency list

```
0: 1, 2
1: 0, 2
2: 0, 1, 3
3: 2
```

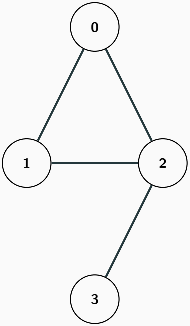
```
vector<int> adj[4];
adj[0].push_back(1);
adj[0].push_back(2);
adj[1].push_back(0);
adj[1].push_back(2);
adj[2].push_back(0);
adj[2].push_back(1);
adj[2].push_back(3);
adj[3].push_back(2);
```



Adjacency matrix

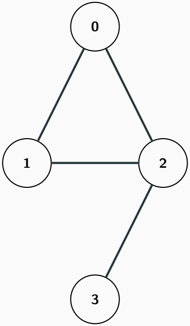
```
0 1 1 0
1 0 1 0
1 1 0 1
0 0 1 0
```

```
bool adj[4][4];
adj[0][1] = true;
adj[0][2] = true;
adj[1][0] = true;
adj[1][2] = true;
adj[2][0] = true;
adj[2][1] = true;
adj[2][3] = true;
adj[3][2] = true;
```



Edge list

```
0, 1
0, 2
1, 2
2, 3
```




```
vector<pair<int, int> > edges;
edges.push_back(make_pair(0, 1));
edges.push_back(make_pair(0, 2));
edges.push_back(make_pair(1, 2));
edges.push_back(make_pair(2, 3));
```

Efficiency

	Adjacency list	Adjacency matrix	Edge list
Storage	$O(\lvert V\rvert + \lvert E\rvert)$	$O(\lvert V\rvert^2)$	$O(\lvert E\rvert)$
Add vertex	$O(1)$	$O(\lvert V\rvert^2)$	$O(1)$
Add edge	$O(1)$	$O(1)$	$O(1)$
Remove vertex	$O(\lvert E\rvert)$	$O(\lvert V\rvert^2)$	$O(\lvert E\rvert)$
Remove edge	$O(\lvert E\rvert)$	$O(1)$	$O(\lvert E\rvert)$
Query: are u, v adjacent?	$O(\lvert V\rvert)$	$O(1)$	$O(\lvert E\rvert)$

- Different representations are good for different situations

Example problem: Grandpa Bernie

- https://open.kattis.com/problems/grandpabernie